JOURNAL OF PROPULSION AND POWER
Vol. 20, No. 5, September–October 2004

Preliminary Fan-Blade Design Using Intermediate Response Approximations

J. F. Worgan* and T. M. Edmunds†
Rolls-Royce plc, Filton, England BS34 7QE, United Kingdom
and
A. J. Keane‡
University of Southampton, Southampton, England SO17 1BJ, United Kingdom

A midrange approximation method for design optimization is formulated. This utilizes intermediate response variables, amenable to simple polynomial representation, to provide a reasonable approximation over a trust region. It has applications to computationally expensive, nonlinear optimization problems where a designer's knowledge may be exploited to select appropriate intermediate variables. Application to the preliminary structural design of a fan blade is illustrated in two case studies; results for a global optimization strategy and a direct search have been included for comparison. Significant improvement in the number of function calls necessary to identify a local optimum is demonstrated in the case studies considered.

Introduction

ASSESSMENT of the performance and reliability of a particular design concept requires execution of complex simulation models. Direct coupling of these to an optimization algorithm is often impractical due to the prohibitive computational cost,[1] and hence much research has focused on the use of approximation models as surrogates; a literature review in this field is provided by Vanderplaats.[2] A critical factor in the success of these strategies is the accuracy of the model over its region of application. This region size is typically classified as local, global, or midrange.[3]

Local models commonly employ a low-order Taylor-series expansion of the optimization functions, based upon the function value and sensitivities at the current design point.[4] Domain knowledge is often utilized to extend its range of validity, by appropriate transformation of the design variables,[5] or the introduction of intermediate response functions.[6,7] Move limits may be imposed to restrict the step size to a subregion where the model is regarded as accurate.[8] A new approximation model is fitted for each optimization loop, repeating until any convergence criteria are met.

Alternatively, global models may be employed to approximate the system behavior over the whole design space. They may be based upon a simplified simulation model of the system[9] or a response surface model generated from the function values and in some cases sensitivities at a number of data points.[10] The latter are often based upon low-order polynomials fitted to the data set by regression, enabling a reduction in numerical noise.[11,12] Where the underlying functions are multimodal, functions that are capable of describing multiple extrema have been used: for example, radial basis functions[13] and Kriging.[14] In some approaches the functional form is also determined from the data, for example, genetic programming[15] and neural networks.[16]

The third approach is to utilize a simpler, commonly low-order polynomial-response-surface model and restrict the step size to a subregion of the design space (midrange approximation).[17] In trust-region algorithms,[18] this step size (trust radius) is systematically updated by how well the approximation model predicts the improvement in the objective or penalized objective.

The method presented here is based on the midrange method formulated by Toropov et al.[19] and Van Keulen and Toporov.[20] It is adapted to incorporating intermediate response variables that are amenable to representation by low-order polynomial expressions and that form the basis of a set of explicit formulae for the optimization functions. By exploiting the designer's knowledge to select these variables, it is anticipated that fewer analyses will be required to identify an optimal design, reducing the time and cost associated with the design search.

To illustrate this approach two preliminary fan-blade design problems are considered here: optimizing the aerofoil shape to minimize mass, subject to frequency constraints, and choice of the aerofoil stack to minimize the steady stress.

Intermediate Response Variables

Intermediate response variables (IRVs) are effective when a problem can be decomposed into two parts: an implicit calculation of the response variables, which are amenable to low-order polynomial approximation, and a subsequent explicit calculation of the nonlinear optimization functions.

Levenberg[21] applied this approach to the solution of the least-squares problem, where the residual between the real and approximate functions at each data point was selected as an IRV, represented by a first-order Taylor series expansion. The nonlinear objective function, the sum of the squared residuals, is a simple explicit function of these variables and a trust region constraint was imposed to restrict the step size to a region where the approximation is valid.

In structural design, Vanderplaats and Salajegheh[6] and Kodiyalam and Vanderplaats[7] demonstrated that a significant improvement in the approximate stress prediction could be achieved by utilizing an intermediate approximation model of the nodal forces. This is simply illustrated by the following truss design problem: select the member area A_j to meet the stress constraint,

$$\sigma_j \leq \sigma_{\text{allowable}} \tag{1}$$

The member stress is inversely proportional to the member area, but employing the force approximation Φ proposed by Vanderplaats and Salajegheh[6] and Kodiyalam and Vanderplaats,[7] this constraint may

Received 14 April 2003; revision received 4 May 2004; accepted for publication 10 May 2004.

*Team Leader Mechanical Methods Bristol, Design Technology and Mechanical Methods, P.O. Box 3, Gypsy Patch Lane.

†Company Specialist Advanced Methods (Retired), Design Technology and Mechanical Methods, P.O. Box 3, Gypsy Patch Lane.

‡Professor and Chair of the Computational Engineering and Design Group, School of Engineering Sciences, Highfield.

be rewritten as a linear function of A_j,

$$\sigma_j = [\Phi_j(A_k) + \Phi'_j(A_k)(A_j - A_k)]/A_j \le \sigma_{\text{allowable}}$$

$$A_j - [\Phi_j(A_k) + \Phi'_j(A_k)(A_j - A_k)]/\sigma_{\text{allowable}} \ge 0 \qquad (2)$$

Vanderplaats and his coworkers have demonstrated the application of this method to the design of frames and three-dimensional continuum structures.

This paper explores the use of IRVs in minimizing the peak steady stress in a component and a frequency-tuning problem.

Multipoint Approximation Method

The following section outlines the multipoint approximation method formulated by Toropov et al.[19] and Van Keulen and Toporov,[20] with the modifications necessary to utilize IRVs emphasized.

Optimization Problem

The optimization problem is formulated as follows:
Minimize

$$F(\bar{x}), \qquad \bar{x}_L \le \bar{x} \le \bar{x}_U \qquad (3)$$

subject to

$$G_j(\bar{x}) \le 0, \qquad j = 1, m$$

$$H_k(\bar{x}) = 0, \qquad k = 1, p$$

Midrange Approximation Method

In the midrange approximation method, the single optimization iteration is replaced by a sequence of optimization iterations controlled by an outer loop. It is this outer loop that controls the assessment of the quality of the approximation model, performs the convergence check, and, if appropriate, defines the approximation models and search region for the next inner optimization cycle.

In the original method an approximation model is generated for the objective and each constraint equation. To utilize IRVs the inner loop is modified as follows:
minimize

$$F[x, \tilde{R}(\bar{x})], \qquad \bar{A} \le \bar{x} \le \bar{B} \qquad (4)$$

subject to

$$G_j[\bar{x}, \tilde{R}_l(\bar{x})] \le 0, \qquad j = 1, m \qquad l = 1, r$$

$$H_k[\bar{x}, \tilde{R}_l(\bar{x})] = 0, \qquad k = 1, p \qquad l = 1, r$$

Here $\tilde{R}$ indicates the approximate intermediate response functions, and A and B indicate the upper and lower move limits, which prescribe the boundaries of the current trust region.

The approximation models are based on a polynomial response surface model of the form

$$\tilde{R} = c_o + c_i x_i + c_{ij} x_i x_j, \qquad i = 1, n \qquad j = 1, n \qquad (5)$$

A weighted least-squares approach is used to determine the polynomial coefficients. Where insufficient data points exist, the coefficients of the quadratic terms are effectively set to zero by assignment of low weights.

The weights assigned to each data point are based on proximity to the center of the trust region, value of the objective function, and proximity to a constraint boundary. Inclusion of IRVs prevents the application of constraint-specific weightings, exploited in the original method,[19,20] but has the incidental advantage that a single weight matrix may be formed to determine the coefficients of the response approximations.

At each data point x_k in the current design history, the weight w_k is defined by

$$w_k = w_k^s w_k^o w_k^c \qquad (6)$$

Points within or in close proximity to the current trust region are included in the response-surface fit, and hence are assigned a nonzero weight. This method is a multipoint approximation method because points from previous iterations may be included in the response surface model:

$$w_k^s = 1.0, \qquad \Delta N_l \le x_k \le \Delta N_u$$

$$w_k^s = \cos\left\{[x_k - (\Delta B_l)] \times \frac{\pi}{2(\Delta N_l - \Delta B_l)} - \frac{\pi}{2}\right\} \qquad \Delta B_l \le x_k \le \Delta N_l$$

$$w_k^s = \cos\left[-(x_k - \Delta B_u) \times \frac{\pi}{2(\Delta N_u - \Delta B_u)} - \frac{\pi}{2}\right] \qquad \Delta N_u \le x_k \le \Delta B_u$$

$$w_k^s = 0, \qquad x_k \le \Delta B_l, \qquad \Delta B_u \le x_k$$

$$\Delta B_l \le \Delta N_l \le A \le B \le \Delta N_u \le \Delta B_u \qquad (7)$$

ΔN_l and ΔN_u define a region (neighborhood) encompassing the current trust region [default extents $= 1.5 \times (A, B)$].

A new blend region is introduced here, ΔB_l and ΔB_u, which prescribes a diminishing weight function at the boundary of the trust region based on a cosine function. The default extents of this blend region are $2 \times (A, B)$.

Points with a low objective value are given higher weight in the response-surface fit to improve the approximation in the region where the local optimum is considered most likely to lie:

$$F_{T\max} = \max[F(x_k)], \qquad F_{T\min} = \min[F(x_k)]$$

$$F_{\max} = \max\left[N \times \frac{(F(x_k) - F_{T\min})}{(F_{T\max} - F_{T\min})}\right]$$

$$F_{\min} = \min\left[N \times \frac{(F(x_k) - F_{T\min})}{(F_{T\max} - F_{T\min})}\right]$$

$$w_k^o = 0.25 + 0.75 \times \frac{(F_{\max} - F(x_k))}{\left(F_{\max} - F_{\min} + 1.0 \times 10^{-12}\right)} \qquad (8)$$

On the basis that the optimum is likely to be situated on a constraint boundary, points that have a constraint close to criticality are given more weight:

$$G_{j_{T\max}} = \max[G_j(x_k)], \qquad G_{j_{T\min}} = \min[G_j(x_k)]$$

$$G_{\min} = \min\left[\frac{G_j(x_k)}{G_{j_{T\max}} - G_{j_{T\min}}}\right]$$

$$w_k^c = \frac{3.0 - 4.0 \times G_{\min}}{3.0}, \qquad |G_{\min}| \le 0.5$$

$$W_k^c = \frac{1.0}{3.0}, \qquad |G_{\min}| \ge 0.5 \qquad (9)$$

The default size of the first trust region is defined to be 0.2 times the permissible design space. In problem-specific applications this region may be sized based upon knowledge of the likely range of applicability of a linear approximation model.

This region encompasses the initial design point or "anchor." The anchor point is positioned in the corner of the trust region, initially a distance of 5/21 times the trust region size from the lower design variable bounds. At subsequent iterations the selection of the location of the anchor point depends upon the previous movement history.

A plan analysis is performed prior to the first optimization iteration to form the basis of the first linear approximation model. This plan entails an analysis at the anchor point and a further N analyses at data points generated by perturbing each design variable in turn by a step of size 10/21 times the trust-region size.

On completion of the inner optimization loop, an exact analysis is executed at the final solution x_0 and the results are characterized by the quality of the objective, constraint, and response predictions, the location of the point, the current trust region size, and the movement history. The move limit strategy developed by Toropov et al.[19] and Van Keulen and Toporov[20] was employed in this implementation.

Convergence is achieved when the optimum of the approximate problem exists within a small trust region and the quality of the approximation model is high at the predicted optimum. This method is effectively a local search strategy and hence there is no guarantee that the optimum identified is global. A restart strategy could be adopted to improve the global nature of the search.

An augmented Lagrangian method[22] is used to solve the optimization problem in the inner loop; constant convergence criteria are applied to each inner optimization study.

Case Studies

In this section two case studies are presented to illustrate the application of this technique in fan-blade design.

Case Study 1: Blade Leaning

In rotor-blade design, the initial aerodynamic streamline sections are stacked radially on a line passing through the section centroids. The maximum steady stress over the aerofoil surface may usually be reduced by offsetting the sections from the radial line, away from the direction of rotation. This "lean" effectively alters the centrifugal force distribution to counterbalance the effect of the aerodynamic pressure distribution.

A three-dimensional finite element model of the blade and attachment is required to accurately model the aerofoil, but an aerofoil-only model has been used in each case study to demonstrate the basic concepts. A mesh-morphing approach was adopted to prevent noise in the calculated results due to changes in the finite-element discretization error.

Design variables were assigned to four streamline sections, describing circumferential rotation about the engine axis from hub to tip; values at intermediate sections were determined by cubic interpolation and the design-variable range was standardized to $[-1, 1]$. The objective is to minimize the absolute value of the worst principal stress over the aerofoil surface. As the blade sections are perturbed the worst principal stress changes location, sign, and direction, leading to a number of discontinuities in the objective function; this is illustrated by the parameter study in Fig. 1. However, as Fig. 2 indicates, the constituent stress components at each surface node are essentially linear over the design-variable range and hence amenable to linear approximation and use as IRVs.

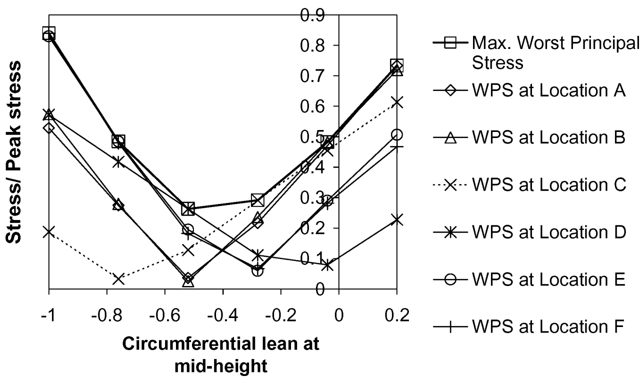

Fig. 1 Compound objective function: maximum worst principal stress.

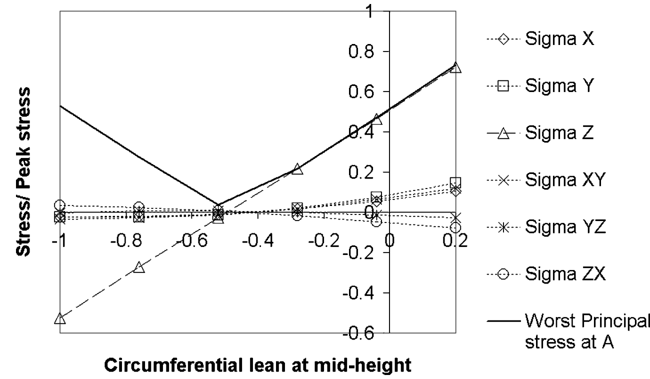

Fig. 2 Constituent stress components at point A.

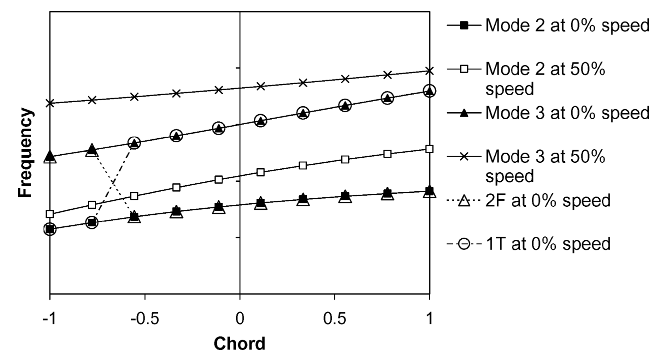

Fig. 3 Change in frequency as the chord at midheight varies.

Case Study 2: Frequency Tuning

Case study 2 concerns the optimization of the mass of a rotor-blade aerofoil subject to the frequency constraints imposed by resonance and flutter avoidance. The aerofoil chord was varied at three streamline sections and the maximum thickness at five streamline sections from hub to tip. Side constraints were imposed on these variables to restrict the feasible design space to a region where the aerodynamic performance was acceptable and each design variable was standardized to the range $[-1, 1]$.

Resonance criteria were imposed by requiring adequate separation between the primary modes and the required engine-order lines at a number of speeds in the running range. An engine-order line depicts the change in frequency with rotor speed for a frequency that occurs at an integral number of revolutions of the rotor. The required engine orders are defined by the potential sources of excitation: intake distortion, the support struts, and the adjacent blading. Simple coupled flutter constraints were imposed by requiring adequate separation between the flexural and torsional modes over the running range.

The mode shapes are characterized by comparing the modal displacements with a set of datum cantilever mode shapes. The error between the cantilever and the actual mode shape for a given mode is defined here as the mode characterization function.

In contrast to the first case study, the optimum of this problem lies on a constraint boundary. These constraints are non linear and, where crossing modes are a problem, can exhibit significant slope discontinuities. Crossing mode behavior is observed when a flexural mode has a frequency lower than but close to a torsional mode under static conditions. The centrifugal stiffening effect will cause the modes to approach and even appear to cross.

Figures 3 and 4 illustrate a design sweep that exhibits this phenomenon. Figure 3 illustrates the variation in mode and the characterized mode and Fig. 4 the change in mode-shape characterization function with design-variable change.

Based on these results the frequency and the mode-shape characterization function were selected as IRVs. An additional constraint was also included to keep the first torsional mode below the second flexural mode.

Results

The following search strategies were compared for each case study:

Analysis A: midrange approximation method using IRVs.

Analysis B: midrange approximation method using the optimization functions. This method differs from the method developed by Toropov et al.[19] and Van Keulen and Toropov,[20] because it does not exploit the constraint specific weightings in the regression.

Analysis C: global-polynomial-response-surface model based on IRVs. In Case Study 1 a first-order polynomial plus squared terms is used, based on a nine-point design of experiment plan (design-space center and center of the bounding faces), whereas in Case Study 2 a second-order polynomial, fitted by regression to 500 points from a Latin hypercube, is used. The selected optimizer [sequential quadratic programming method (DONLP)] is provided by the commercial package iSIGHT, Engineous Software Inc.

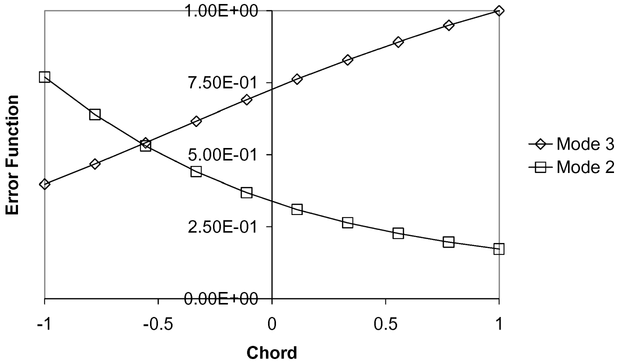

Fig. 4 First torsion mode characterization illustrating crossing modes.

Analyses D and E: global-response-surface model of the optimization functions using a quadratic polynomial and a radial basis function (cubic spline), respectively. The initial dataset is based on a full-factorial array in Case Study 1 and the 500 points generated by the Latin hypercube in case study 2. The genetic algorithm in the University of Southampton code OPTIONS was used in these analyses.

Analysis F: sequential quadratic programming.

A Sun Ultra 30 (248 MHz Sparc processor, 128 Mbyte RAM, 1-Gbyte swap, SUN OS 5.5.1) was used in this study.

Tables 1 and 2 contain a summary of the optimization results for each case study. Each analysis has been assigned a rank for design quality, elapsed time, and the number of function evaluations. The third rank will become more important as the cost of the finite element analysis increases.

Case Study 1

Analysis A outperforms the other methods, identifying a good design (within 0.2% of "best" (Analysis F)) in the lowest elapsed time and analysis count. This is due to the quality of the initial linear

Table 1 Case Study 1: Optimization History

Variable	Analysis A	Analysis B	Analysis C	Analysis D	Analysis E	Analysis F
No. of design variables	4	4	4	4	4	4
No. of response	3277	1	3277	1	1	0
No. of function evaluations	15	82	10	257	257	251
			Final Design			
Section 1	0.0000	0.0000	0.0000	0.0000	0.0000	0.00000
Section 6	−0.0522	−0.0707	−0.0195	−0.1631	−0.1749	−0.0551
Section 11	−0.4414	−0.4014	−0.3838	−0.3912	−0.4684	−0.4388
Section 16	−0.7332	−0.7640	−0.5874	−0.9361	−0.8424	−0.7261
Section 21	−0.9363	−0.7130	−0.9997	−0.9999	−0.7999	−1.0000
Final objective (stress/peak stress)	.1745	.1815	.2009	.2464	.2249	.1742
Elapsed time	10.5 min	20 min	42 min	53 min	53.5 min	46 min
			Ranking			
Quality	2	3	4	6	5	1
Elapsed time	1	2	3	5	6	4
Function calls	1	3	2	5	5	4

Table 2 Case Study 2: Optimization History

Variable	Analysis A	Analysis B	Analysis C	Analysis D	Analysis E	Analysis F
No. of design variables	8	8	8	8	8	8
No. of response	19	412	19	412	85	0
No. of constraints	411	411	411	411	84	411
No. of function evaluations	61	321	501	501	501	495
			Final design			
Max T S1	0.5116	0.0452	0.9968	−0.0090	0.0414	0.3923
Max T S6	−0.2537	−0.3245	1.0000	0.0747	0.0013	−0.3785
Max T S11	−0.5650	−0.4999	−0.9928	−0.1626	−0.3899	−0.5839
Max T S16	−0.6050	−0.6927	−0.4042	−0.2955	−0.4978	−0.6502
Max T S21	−1.0000	−0.8809	−0.3765	−1.0000	−0.6707	−0.9617
Chord S1	−0.4964	−0.8485	1.0000	−0.9993	−0.9374	0.2196
Chord S11	−0.3374	−0.9865	−0.9999	−1.0000	−0.9974	−0.8479
Chord S21	0.8000	0.9000	1.0000	1.0000	0.6000	−1.0000
No. of constraint violations	4	0	8	5	0	0
Worst constraint	−0.01	N/A	−0.199	−0.07	N/A	N/A
Final objective	1.0368	0.9452	1.2896	1.1583	1.0699	0.9756
(minimum) elapsed time	13 h 28 min	13 h 53 min	20 h 31 min	20 h 16 min	21 h 18 min	21 h 23 min
			Ranking			
Quality rank	4	1	6	5	3	2
Elapsed time	1	2	4	3	5	6
Function calls	1	2	4	4	4	3

models of the IRVs over a significant percentage (80%) of the design space. The time saving is not as significant as may be anticipated as a result of the low analysis count because 73% of the analysis time is spent in the inner optimization loop. This is due to the requirement to evaluate the objective function at each iteration and the number of iterations being performed to achieve convergence within the inner optimization study. A staged convergence strategy may address the latter. Analysis B provides the next best design, again in a shorter time than F, illustrating that the midrange strategy enables a simple polynomial model to capture the nonlinear slope-discontinuous objective function. Analysis C, based on a global approximation model of the IRVs, achieves an adequate design in very few function calls, emphasizing the quality of the linear model over the design space, shown in analysis A. The time comparison is not relevant here because it reflects the costs associated with loosely coupling the analysis and optimization code (software start-up, file-parsing). The global approximation models of the optimization functions fail to generate an acceptable design.

Case Study 2

Analysis A outperforms the other analyses with regard to analysis time and the number of function evaluations, but fails to identify the best design. The latter is identified within the design history in Analysis B. However, B failed to converge to this design point, terminating at the maximum number of design iterations (40), with the solution oscillating about the constraint boundary. The next best design was identified by direct SQP search; however, this required the longest analysis time of all the approaches adopted. Analysis E also converges to a feasible design, the heaviest of the three, but again the computational cost of the function calls is high. The global polynomial approximation models based on IRVs or optimization functions failed to identify an acceptable design.

The choice of the best method is less clear for this case study. However, the midrange approximation strategies provide a tool for design improvement in a significantly reduced timeframes compared to the other strategies.

Conclusions

A midrange approximation method has been developed that utilizes IRVs, amenable to representation by a low-order polynomial, to provide cheap and accurate approximation models in the design search. In the case studies presented, this method reduces the number of analyses required in the design search, when compared to the global approximation strategies and the direct search. The reduction in computer elapsed time is not significant here, due to the time associated with evaluating the optimization functions in the inner optimization loop. However, for more complex finite element models, the reduction in the number of analyses will yield a significant time benefit. The success of this method is based upon the inclusion of the designer's knowledge in simplifying the search process and hence is not a "black box" approach. However, for component-specific design systems, this approach offers the potential to include optimization as a routine part of the design process, while meeting the reduction in design times driven by a competitive marketplace.

References

[1]Sobieszczanski-Sobieski, J., "Multi-Disciplinary Design Optimization (MDO) Methods: Their Synergy with Computer Technology in Design Process," *Proceedings of the Royal Aeronautical Society Conference on Multi-Disciplinary Design and Optimisation*, Royal Aeronautical Society, London, 1998, pp. 11.1–11.14.

[2]Vanderplaats, G. N., "Structural Design Optimization Status and Direction," *Journal of Aircraft*, Vol. 36, No. 1, 1999, pp. 11–20.

[3]Barthelemy, J. F. M., and Haftka, R. T., "Approximation Concepts for Optimum Structural Design—A Review," *Structural Optimization*, Vol. 5, 1993, pp. 129–144.

[4]Haftka, R. T., and Gürdal, Z., *Elements of Structural Optimization*, 3rd ed., Kluwer Academic, Dordrecht, The Netherlands, 1992, pp. 211–219.

[5]Schmidt, L. A., and Farshi, B., "Some Approximation Concepts for Structural Synthesis," *AIAA Journal*, Vol. 12, No. 5, 1974, pp. 692–699.

[6]Vanderplaats, G. N., and Salajegheh, E., "New Approximation Method for Stress Constraints in Structural Synthesis," *AIAA Journal*, Vol. 27, No. 3, 1989, pp. 352–358.

[7]Kodiyalam, S., and Vanderplaats, G. N., "Shape Optimization of Three-Dimensional Continuum Structures via Force Approximation Techniques," *AIAA Journal*, Vol. 27, No. 9, 1989, pp. 1256–1263.

[8]Lust, R. V., and Schmidt, L. A., "Alternate Approximation Concepts for Space Frame Synthesis," *AIAA Journal*, Vol. 24, No. 10, 1986, pp. 1676–1684.

[9]Alexandrov, N. M., Lewis, R. M., Gumbert, C. R., Green, L. L., and Newman, P. A., "Optimization with Variable-Fidelity Models Applied to Wing Design," AIAA Paper 2000-0841, Jan. 2000.

[10]Myers, R. H., and Montgomery, D. C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, Wiley, New York, 1995, pp. 1–10.

[11]Papila, M., and Haftka, R. T., "Response Surface Approximations: Noise, Error, Repair, and Modeling Errors," *AIAA Journal*, Vol. 38, No. 12, 2000, pp. 2336–2343.

[12]Haim, D., Giunta, A. A., Holzwarth, M. M., Mason, W. H., Watson, L. T., and Haftka, R., "Comparison of Optimization Software Packages for an Aircraft Multidisciplinary Design Optimization Problem," *Design Optimization: International Journal for Product and Process Improvement*, Vol. 1, No. 1, 1999, pp. 9–23.

[13]McDonald, D. B., Grantham, W. J., and Tabor, W. L., "Response Surface Model Development for Global/Local Optimization Using Radial Basis Functions," AIAA Paper 2000-4776, Sept. 2000.

[14]Sacks, J., Welch, W. J., Mitchell, T. J., and Wynn, H. P., "Design and Analysis of Computer Experiments," *Statistical Science*, Vol. 4, No. 4, 1989, pp. 409–435.

[15]Alvarez, L. F., Hughes, D. C., and Toropov, V. V., "Optimization of the Manufacturing Process for Roman Cement," *Proceedings of the 2nd ASMO UK/ ISSMO Conference on Engineering Design Optimization*, University of Wales, Swansea, 2000, pp. 27–32.

[16]Javadi, A. A., and Farmani, R., "A Hybrid Optimization Method Based on Neural Network and Genetic Algorithm," *Proceedings of the 3rd ASMO UK/ISSMO Conference on Engineering Design Optimization*, University of Leeds, Leeds, England, U.K., 2001, pp. 127–132.

[17]Schoofs, A. J. G, van Houten, M. H., Etman, L. F. P., and van Campen, D. H., "Global and Mid-Range Function Approximation for Engineering Optimization," *Mathematical Methods of Operations Research*, Vol. 46. No. 3, 1997, pp. 335–359.

[18]Alexandrov, N., Dennis, J. E., Jr., Lewis, R. M., and Torczon, V., "A Trust Region Framework for Managing the Use of Approximation Models in Optimization," NASA CR-201745, ICASE Rept. 97-50, Oct. 1997.

[19]Toropov, V. V., Filatov, A. A., and Polynkin, A. A., "Multiparameter Structural Optimization Using FEM and Multi-Point Explicit Approximations," *Structural Optimization,* Vol. 6, 1993, pp. 7–14.

[20]Van Keulen, F., and Toropov, V. V., "New Developments in Structural Optimization Using Adaptive Mesh Refinement and Multi-Point Approximations," *Engineering Optimization*, Vol. 29, 1997, pp. 217–234.

[21]Levenberg, K., "A Method for the Solution of Certain Non-linear Problems in Least Squares," *Quarterly Journal of Applied Mathematics*, Vol. 2, No. 2, 1944, pp. 164–168.

[22]Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design: With Applications*, McGraw–Hill, New York, 1984, Sec. 5.8.